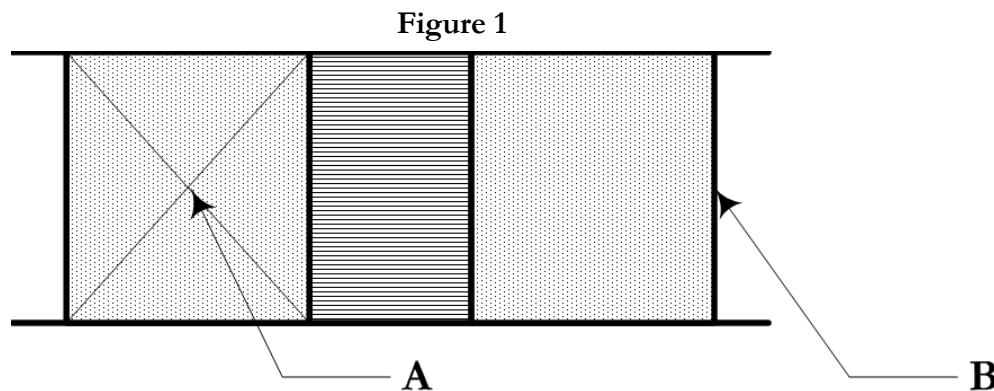


## The Change of Resistance (COR) Method for Determining the Average Steady-state Temperature of an Electrical Conductor

The Change of Resistance (COR) method is used for determining the average steady-state temperature of an electrical conductor during operation, through a series of electrical resistance measurements over time. In contrast, a thermocouple, by definition will give a reading at the point where the thermocouple is applied.

Consider the Figure 1, below, a representation of the cross section of a bobbin wound motor coil on an iron lamination core:



Consider that the hottest part of the coil is at the geometric center (A) where the heat sinking to surrounding surfaces or media is minimized and the heat sourcing contribution of surrounding conductors is maximized. The COR method will provide the average temperature of the entire conductor that forms the coil. The thermocouple, applied necessarily to the surface of the coil (B), where the heat sinking to the surrounding air is maximized and the contribution of the heat sourcing mass is from a single direction, will always be at a lower temperature.

The COR method leverages the fact that the electrical resistance of a metallic conductor is linearly proportional to its temperature. A series of resistance values are taken as the system cools, and these resistance values are then converted to corresponding temperature values. An initial temperature is calculated by extrapolating a curve fit of the data points to time zero (the moment when power was removed from the coil and cooling began). The COR method is best used when the electrical conductor is physically inaccessible, preventing the application of thermocouples or other means of direct temperature measurement at the conductor surface.

Though in principle a relatively simple approach to measuring coil temperature, the COR method can give inaccurate results if one or more elements of the method are performed improperly or if assumptions that the method makes are not appropriate for the specific system under test. The issues described below may easily lead to errors of 20-30% or more in determining the temperature at time  $t = 0$  (defined as the time when power is removed from the conductor and the onset of cooling begins).

## Issues with the COR Measurement

### Lack of constant offset –

The cooling of a conductor over time is characterized by one or more exponential decay functions. Since the governing equations contain a constant offset and assume that cooling begins at time zero, direct application of an exponential curve fitting operation without close attention to the fitting parameters can lead to significant error. Typically most curve fitting software functions use the following exponential fit:

$$y(x) = Ae^{-x}$$

Note that this equation lacks a linear offset term, which is needed to offset the ambient temperature  $T_a$  for the cooling equation. Ignoring the offset contributed by  $T_a$  will always result in a poor exponential fit that results in a value at time zero far lower than the actual value.

This curve fitting function can be used, provided a relative temperature (indexed to  $T_a$ ) is instead used. This is obtained by subtracting  $T_a$  from each temperature value in the curve, resulting in the proper curve fit.

### Improper mathematical fit –

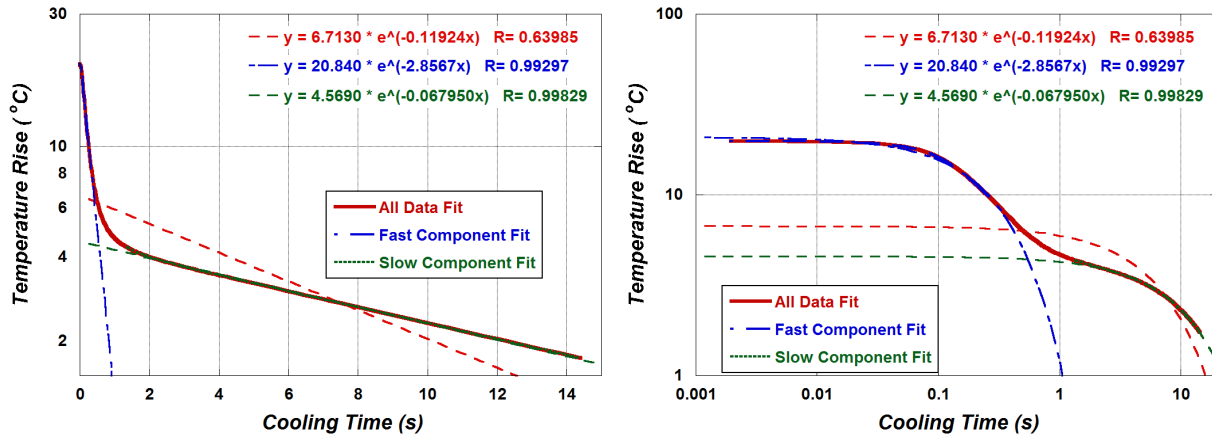
The use of the exponential fit with constant offset has been proven through both underlying thermodynamic science as well as through extensive empirical testing, to most closely model the coil cooling phenomena. Some have proposed using polynomial (most typically fourth order polynomial) fits. The justification of this method is that since a higher-order polynomial, namely a fourth-order polynomial, allows for more degrees of freedom, it is anticipated that such a polynomial fit will enable improved fitting that will be more immune to issues encountered with the exponential fitting, potential secondary exponential components, and a need to address offsets to the data (such as compensating for ambient temperature). However, the reliability of the ability of a fourth-order polynomial to be used for extrapolating the initial temperature breaks down as the duration between the onset of cooling and the beginning of data collection increases.

The results of experimental testing show that despite an apparent good fit for the fourth-order polynomial fit, the quality of the fit breaks down as the demand for extrapolation increases. This is due to the fact that the fourth-order polynomial is being force-fit to the data set through its many degrees of freedom. The exponential decay model, by contrast, fits by matching closely to the physical model. The fact that the exponential model follows natural behavior removes a need for as many degrees of freedom as are offered in polynomial fits, and therefore tends to maintain accuracy as the start of the curve deviates away from time zero.

## Comparison of Model to Experimental Data

Consider that actual product or test samples are not formed of a homogeneous mass but rather several masses each with different specific heats and different thermal conductivities, i.e., the mass of the coil itself, the mass of the iron core, the mass of the enclosure, etc. Testing with real such “systems” show prevalence for complex cooling, which from the experience with the modeling, suggests that parallel cooling paths exist in most real-world systems. The deviation from a straight line (when plotted in a semilog plot) during the first few hundred seconds of cooling demonstrates that the entire cooling process, and most importantly, the cooling nearest time zero, does not follow a single exponential rate. The real-world data show a “fast cooling” process which then yields to a “slow cooling” process, likely attributable to cooling of thermal masses with relatively small and large heat capacities and/or thermal resistances, respectively. Figure 2, below illustrates the break point between the “fast” and “slow” cooling components of the thermal decay of the overall product. The left is a linear-log view, illustrating the different slopes for each exponential term while the right is a log-log plot allowing more convenient observation of the two cooling components of the overall thermal decay function.

Figure 2



As was shown with the modeling results, approximating more complex cooling with a single exponential term will result in unacceptably large curve fitting errors and therefore a fitting algorithm that uses a single exponential term cannot be used. However, the data shows that the cooling rate can be well approximated near time zero by only two terms (three or more exponents are not expected to add more accuracy, and would be expected to add too many degrees of freedom to the curve fit, where extrapolation to time zero would more likely give unreliable results). Therefore, the recommended fitting algorithm is based on a fit that utilizes two exponential terms.

### Description of the Fitting Algorithm

The results of the real-world tests show that an accurate curve fit around time zero would be best approximated by a sum of two exponential terms, which is a balance of enabling capture of the change in cooling rates that is often observed immediately after time zero and an avoidance of giving too many degrees of freedom to the curve fitting algorithm (which destabilizes the ability to use the resulting fit for extrapolation to time zero). Therefore, the curve fitting algorithm uses the following general form:

$$\Delta R(t) = Ae^{-\alpha t} + Be^{-\beta t} + C$$

where  $\Delta R$  is the change in resistance above the room temperature resistance, and  $A$ ,  $B$ ,  $\alpha$ ,  $\beta$  and  $C$  are fitting parameters. It is known that sums of exponentials tend to be poorly bounded and are not conducive to converging around a unique solution due to the considerable degrees of freedom in the fitting parameters. However, it is visually observable from the modeling and real-world data when one exponential term takes over dominance from another, particularly in the semilog and log-log plots. Therefore, the cooling curve is first analyzed to identify this point in time where the cooling rate changes.

The real-world data suggests that, in the beginning, the cooling is somewhat complex, likely a result of both exponential terms, then moves towards cooling that is dominated by a single exponential term. The fitting strategy therefore first locates and then fits this region where a single exponential term dominates, solving for one of the two exponential terms. In this region, the fitting equation simplifies to the following:

$$\Delta R(t) = Be^{-\beta t} + C$$

Once the “linear” region has been fit and three of the five terms in the fitting equation have been identified, the focus is now in solving for the remaining terms in the region nearest time zero. This is approached again by simplifying the fitting equation to a single exponential term, this time by subtracting the solved exponential decay from the unknown component:

$$\Delta R(t) - Be^{-\beta t} - C = Ae^{-\alpha t}$$

The result of this subtraction is a cooling curve containing only the unknown exponential term. This result is then fit and the two remaining unknown fitting parameters  $A$  and  $\alpha$  are determined.

The solution for time zero is then obtained through solving the original fitting equation at  $t = 0$ :

$$\Delta R(0) = A + B + C$$

### Application of the Fitting Algorithm to Real-World Data

The fitting procedure described in the previous section was finally encoded into software. Encoding the algorithm into software enhances the ability to provide real world error checking and consistency that is difficult or impossible using manual methods. For example, the COR software identifies issues by:

- Eliminating any points that possess a negative resistance value.
- Eliminating any points which would suggest temperatures outside of what would be expected for the test (temperatures less than the ambient temperature, and temperatures above a preselected maximum coil temperature).
- Eliminating any points that indicate warming of the conductor (i.e., increasing resistance with time).
- Begin curve fitting with the point showing maximum resistance for the entire test.

### Data Collection Rate and Timeframe

Experimental results show that cooling behavior often deviates significantly from the ideal single decay case, and the most critical data points may be obtained within the first few seconds of cooling, well before ambient is reached. These cooling rates are also heavily influenced by the physical size of the system, resulting in cooling timeframes which may vary over many orders of magnitude (less than one second to many hours or days). Therefore, a single, constant, rule-of-thumb applicable to all cases is impossible to obtain.

### Current Practices in Data Collection

Current procedures in data collection for COR tend to be based on a fixed number of points, taken at regular intervals. Typically, the procedure is 10 data points collected at 3 to 30-second intervals; the duration selected by a subjective judgment on how “small” or “large” the object is (and may not be consistent among facilities). Historical documentation on the origins of these practices is lacking, but the focus on a constant sampling rate seems to be related to old timekeeping practices used during COR measurements decades ago. This typically relied on a technician with a stopwatch, reading out time intervals, and a second technician manually recording resistance values with pen and paper. In this situation, varying time intervals with respect to cooling rate and obtaining large numbers of data points was extremely difficult or may have contributed to measurement error.

The introduction of automated data acquisition and software-controlled testing has removed these practical limitations. However, current COR measurement procedures continue to adhere to 10 data points taken over a fixed interval. Unfortunately, this lack of data and flexibility in acquisition rate has led to issues in accurately performing the COR measurement. For example, for large thermal masses, cooling may require several hours to fall by any significant amount: acquiring 10 points over a few minutes in this case may simply give ten readings essentially identical to one another. A fit to this set of data would not be expected to fit well to an exponential model. Another example would be one or more outliers being present in the data set, particularly the first couple of points in the curve. In this case it would be unclear whether the potential outlier was indicative of error (either measurement error or due to residual magnetization or back EMF), or part of a “fast cooling” feature that is captured in only one or two data points.

## Determination of Data Collection Rate

There are some guidelines that can be extracted from reviewing the cooling models and fitting results. First is considering the nature of an exponential decay: that the rate of cooling is fastest at the beginning of the event, and changes in temperature become small as the timeframe becomes large. This suggests that the sampling rate could be optimized if it is conducted on a logarithmic scale, where the rate of sampling is fastest towards the beginning of the cooling event, with longer time intervals as time increases. Again, difficult to accomplish with a degree of consistency using manual methods.

Complicating the issue of selecting a data collection rate is the fact that systems will cool at vastly different rates. For example, a very small system may cool to ambient air within a few seconds, but a very large system may take many hours to cool a few degrees. Though the physical size and geometry of a system could be assessed and a data rate then derived, it was determined to be more consistent (in terms of eliminating any subjectivity) to use a “one size fits all” approach. Though many approaches were considered, the algorithm eventually selected was a “smart” yet not overly complex solution first using the room temperature resistance value and the coefficient of resistance  $K$  to calculate the approximate change in resistance over some temperature interval (for example,  $1^{\circ}\text{C}$ ). The resistance then is constantly monitored, with a new reading recorded only once the resistance has changed by this calculated amount. This results in an optimal number of data points taken.

### *The Four-Probe Measurement*

The Change of Resistance measurement depends heavily on an accurate characterization of the electrical resistance of a conductive element. As this conductor will likely possess a low resistance (a few ohms or less), the resistance of the conductor can be comparable to the resistance of the test leads, connectors, and other measurement equipment. If the measurement technique does not eliminate these additional resistances from the measurement, significant errors can be introduced.

An effective technique in mitigating the effect of resistances from test leads and interfaces is the four-probe, or Kelvin, measurement. This measurement is typically the preferable method for any resistance measurement that utilizes the force-current, measure voltage method. This method is preferred for resistance measurements below approximately  $100\text{ M}\Omega$ . (Above  $100\text{ M}\Omega$ , a two-probe force-voltage, measure-current method is more appropriate.)

Though the four-probe arrangement will give more accurate measurements than the two-probe arrangement, improper four-probe measurement technique can introduce significant errors. One notable error is the practice of connecting the voltage and current probes together to a single common point, such as an alligator clip, then connect this to the test sample. This configuration does remove the effect of lead impedance from the measurement, but leaves the resistance of the alligator clip and the interface between the clip and the test sample. These resistances can be significant when measuring small resistances, as the interface may not be of good electrical quality, may vary with temperature, or some other effect. Corrosion or oxidation on the clip or the surface of the test sample could easily introduce a contact resistance of several ohms, depending on the materials involved. If a convenient means is needed to perform a four-probe measurement is desired, it is necessary to use special Kelvin alligator clips, where the two jaws are electrically insulated, resulting in a true four-probe connections. Current and voltage contacts need to be physically separated from one another, including soldered contacts. Additionally, the addition of relays, switches, and other connection means could have a significant effect on measurement accuracy.

## Summary and Conclusion

- The thermodynamic analysis shows that all cooling, regardless of the system, will exhibit cooling over time that is governed by one or more exponential decay functions. Therefore, all curve fitting procedures should be based on curve fitting algorithms using sums of the base element  $e^{-t}$ .
- It is critical that the correct equation is used to obtain a proper fit of the data. Common fitting errors include a failure to re-normalize the data relative to the ambient temperature. The correct fit will result in the exponential term approaching zero as time approaches infinity. Therefore, a constant offset of the ambient temperature is typically necessary to obtain a proper fit.
- Data rate needs to be conducted on some form of logarithmic time scale, where data collection is fastest at the beginning of cooling, with the interval between data points increasing as the measurement time increases. A convenient automation of this process is to continually monitor the coil resistance and record data when the coil has cooled a predetermined amount (for example, 1°C). This results in a good balance between the number of data points collected, and allows for a single test protocol to be applied to systems with significantly different cooling rates.
- Good measurement practice is critical for this measurement, considering that most measurements will entail conductor resistances of only a few ohms or less. Proper application of the four-probe Kelvin technique is essential.

For the sake of accuracy and consistency both in making the required measurements and in extrapolating to the best approximation of T at  $t = 0$ , in a manner that removes the greatest degree of subjectivity and works equally well for both very large and very small products, the UL developed data acquisition software system COR 4.0 and its successors shall be used for all COR measurements specified in UL Standards. Other methods that yield equivalent results are acceptable but, in any case of discrepancy, the results generated by the COR 4.0 (and its successors) shall prevail.

*Please submit any questions or comments pertaining to this document to [ChangeofResistance@ul.com](mailto:ChangeofResistance@ul.com)*